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The overlap distribution for a non-random frustrated Ising model

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Abstract. In this paper we study the overlap distribution for a non-random two-dimensional frustrated Ising model at low temperatures (the PUD model). We show rigorously that it is a delta distribution. More generally, for any such model that can be reduced to a one-dimensional one, the overlap has necessarily a delta distribution.

1. Introduction

The two basic ingredients of spin glasses are thought to be frustration and randomness. Since the presence of randomness makes the problem very hard, one investigates the concept of frustration on its own [1]. In particular, one looks for which kind of long-range order occurs in deterministic strongly frustrated models [2, 3]. Looking at various frustrated Ising models with periodic interactions in two dimensions, André *et al* [3] found that the long-range order in these systems is always periodic, ferromagnetic or antiferromagnetic and never of the spin-glass type. During the last few years, it has been argued that the overlap distribution function $P(q)$ is a good candidate for a spin-glass order parameter [4]:

$$P(q)=\sum_{\alpha,\beta} p_\alpha p_\beta \delta(q-q_{\alpha,\beta}) \tag{1}$$

where

$$q_{\alpha,\beta}=\lim_{N\to\infty}\frac{1}{N}\sum_i \langle S_i\rangle_\alpha \langle S_i\rangle_\beta . \tag{2}$$

Here α runs over the extremal equilibrium states or pure phases and p_α is the weight of the pure phase $\langle\cdot\rangle_\alpha$, e.g. in the Gibbs state $\langle\cdot\rangle$, i.e. $\langle\cdot\rangle=\Sigma_\alpha p_\alpha\langle\cdot\rangle_\alpha$. A non-trivial $P(q)$ has been given by Parisi for the replica symmetry breaking solution of the SK model [5-7]. (See also the review on spin glasses [8].) Numerical work on a fully frustrated three-dimensional system of classical vector spins revealed, however, that, despite the fact that the long-range order is periodic, the overlap can nevertheless have a non-trivial distribution as expected for real spin glasses [9]. Unfortunately, for a rigorous calculation of $P(q)$ a quite detailed knowledge of the extremal states is needed, together with a natural notion of their weights. This has led us to the piled-up dominos (PUD) model as described in § 2. We repeat a heuristic argument given in [3] to show that this model for sufficiently low temperatures can be reduced to a one-dimensional

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problem, the latter being much simpler. Due to the frustration in this model there are infinitely many ground-state configurations. They represent the extremal phases labelled by α in (1) and the measure p_α can be defined as their weights in the unique state obtained as the zero-temperature limit of the unique translation-invariant finite-temperature equilibrium state. It is interesting to study also the case where the weights are determined by the finite-temperature equilibrium states. The temperature then indicates the interaction-strength dependence.

The main result is that $P_T(q)$ is a Dirac measure concentrated at a non-zero overlap $Q(T)$ slowly varying with T. This result confirms the idea that non-random Ising models with frustration do not exhibit long-range spin-glass order. However, we cannot conclude that this is caused by the deterministic character of the interaction. It is essential that the several phases, which exist strictly speaking only at $T=0$, are not different overall. Reasons for that may be that the interaction is too short ranged or that the lattice dimension is too low or that the one-site configuration space is too small. From our treatment it is clear that, for any model which reduces under specific conditions to a one-dimensional spin model with periodic finite-range interaction, $P(q)$ will have a Dirac distribution.

2. The model

The local Hamiltonians for the PUD model are as follows, for given coupling constants $J, K>0$ and N, M in $\mathbb{N}$:

$$H_{N,M}(x) = -J \sum_{\substack{|j|\leqslant M-1 \\ j \text{ even}}} \sum_{i=-N}^{N-1} x_{i,j}x_{i+1,j} + x_{i,j-1}x_{i,j} + x_{i,j}x_{i,j+1} + K \sum_{\substack{|j|\leqslant M \\ j \text{ odd}}} \sum_{i=-N}^{N-1} x_{i,j}x_{i+1,j}$$

where x takes values in $\{-1, 1\}^{\mathbb{Z}^2}$, the configuration space of a two-dimensional Ising model. The arrangement of ferromagnetic bonds with coupling constants $-J$ and antiferromagnetic bonds with coupling constants K is visualised in figure 1. We restrict

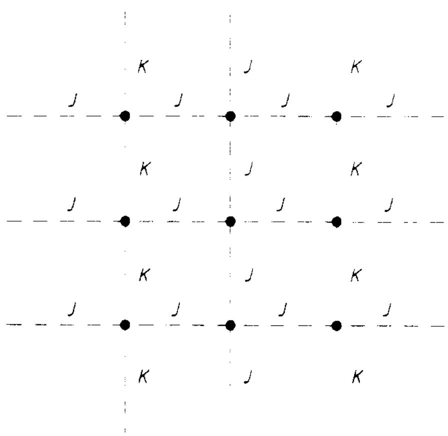

Figure 1. Coupling constants in the PUD model.

our attention to the case where $K \gg J$. One may expect that the antiferromagnetic chains are frozen into one of their two ground-state configurations at sufficiently low temperatures. In almost all ground states of the two-dimensional system, these chains will be ordered in phase as shown in figure 2. Indeed, following the entropy argument of [3], one has to compare the two possible situations for a ferromagnetic chain between two antiferromagnetic ones. When the latter are out of phase, one has only a ferromagnetic chain with two possible ground states while, in the other case, the intermediate chain feels a staggered field of strength $2J$. This causes an infinite degeneracy of the ground state.

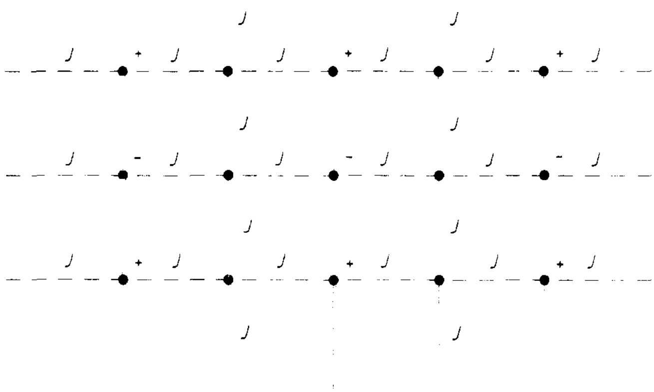

Figure 2. PUD model with $K \gg J$ at very low temperatures.

It is clear that the vertical chains with free spins are completely decoupled near $T=0$. What remains to be studied is the type of ordering in such a chain. We recall that the foregoing arguments are heuristic and that we do not prove that the limits $K \to \infty$ and $T \to 0$ commute. The resulting one-dimensional model, however, can be treated exactly. The effective Hamiltonian is given by

$$H_N^{\text{eff}}(x) = -J \sum_{i=-N}^{N} x_i x_{i+1} - 2J \sum_{i=-N}^{N} (-1)^i x_i$$

where x now takes values in $\{-1, 1\}^{\mathbb{Z}}$. It is convenient to transform this model into a translation-invariant and spin-flip-symmetric one. Therefore we define new variables y_i by

$$(-1)^{[i/2]+[(i+1)/2]} y_i y_{i+1} = x_i$$

where $[z]$ denotes the largest integer smaller than z. If we absorb the factor $2J$ in the inverse temperature, the transformed Hamiltonian is

$$\tilde{H}_N(y) = -\sum_{i=-N}^{N} (y_i y_{i+1} - \tfrac{1}{2} y_i y_{i+2}). \tag{3}$$

This Hamiltonian can be generalised to a set of models with interaction of range n, all of them having a non-zero residual entropy [10, 11] and hence a highly degenerate ground state. In the new variables, the overlap between two configurations y and y' is given by

$$\lim_{N\to\infty} q^N_{y,y'} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{i=-N}^{N} y_i y_{i+1} y'_i y'_{i+1}$$

whenever this limit exists. We remark that one can construct pairs of ground-state configurations for which this limit does not exist, but they are rather rare. Assuming that the equilibrium state ω_β for the model with local Hamiltonians (3) is translation invariant and has local densities ω^N_β, the local overlap distribution is defined by

$$P^N_\beta(q) = \sum_{y_0,\ldots,y_N} \sum_{y'_0,\ldots,y'_N} \delta(q - q^N_{y,y'})\omega^N_\beta(y_0,\ldots,y_N)\omega^N_\beta(y'_0,\ldots,y'_N).$$

This definition includes the case $\beta = \infty$ by taking for ω_∞ the zero-temperature limit of the equilibrium state ω_β. We will show that $\lim_{N\to\infty} P^N_\beta(q)$ exists as a weak limit of measures and tends to a Dirac distribution at $Q(\beta)$ and we will compute the value of $Q(\beta)$. For this purpose we need an explicit expression for the ω^N_β. Their quasiproduct structure is well known [12] and the appropriate transfer-matrix technique allows us to compute the ω^N_β in a straightforward manner [13]. We will generalise (2) to

$$q^N_{x,y} = \frac{1}{2N+1} \sum_{i=-N}^{N} A(x_i,\ldots,x_{i+n})B(y_i,\ldots,y_{i+n})$$

with A and B any pair of observables. In this way we treat a much larger class of models, including the PUD model. The calculation will not depend on the specific form of A or B.

3. The transfer matrix and the overlap distribution

The most typical feature of one-dimensional classical models is that their equilibrium states are completely determined by the n-point correlation functions where n is the range of the interaction [12, 13]. This can be explained as follows. The equilibrium condition requires that the free energy, i.e. internal energy minus entropy, should be minimal. Now the entropy measures the degree of randomness in the state and this will be as large as possible if the direct correlations between the particles are as short as possible. On the other hand, since the interaction is of range n, these correlations extend at least over n points. For one-dimensional models it can be proved that the range of correlation is exactly n [13] and the full state (i.e. all densities) can be reconstructed as the unique so-called quasiproduct state with given n-point density.

For the reduced PUD model, this is done as follows. Let ρ_β denote the three-point function and $\rho'_\beta(x_0, x_1) = \Sigma_{x_2} \rho_\beta(x_0, x_1, x_2)$ the two-point function, then the N-point densities have the following form:

$$\omega^N_\beta(x_0,\ldots,x_N) = \frac{\rho_\beta(x_0,x_1,x_2)\rho_\beta(x_1,x_2,x_3)\ldots\rho_\beta(x_{N-2},x_{N-1},x_N)}{\rho'_\beta(x_1,x_2)\ldots\rho'_\beta(x_{N-2},x_{N-1})}.$$

Because of this structure, these states are called quasiproduct states. The free-energy density of such a state is given by

$$\beta f_\beta(\omega) = \sum_{x_0,x_1,x_2} \rho_\beta(x_0,x_1,x_2)\left(\beta h(x_0,x_1,x_2) + \log\frac{\rho_\beta(x_0,x_1,x_2)}{\rho'_\beta(x_1,x_2)}\right)$$

where $h(x_0, x_1, x_2) = -x_0x_1 + \frac{1}{2}x_0x_2$. Considered as a functional of ρ_β, this expression attains its minimum at

$$\rho_\beta(x_0, x_1, x_2) = \lambda_\beta^{-1}\phi_\beta(x_0, x_1)\psi_\beta(x_1, x_2)\exp[-\beta h(x_0, x_1, x_2)]$$

where ϕ_β and ψ_β are positive eigenvectors of the transfer matrices L_β and L_β^*, respectively, corresponding to their largest eigenvalue λ_β and normalised such that $\Sigma_{x_0,x_1}\phi_\beta(x_0, x_1)\psi_\beta(x_0, x_1) = 1$. The transfer matrix L_β acts on functions $\phi: \{-1, 1\}^2 \to \mathbb{C}$ as follows:

$$(L_\beta\phi)(x_1, x_2) = \sum_{x_0} \exp[-\beta h(x_0, x_1, x_2)]\phi(x_0, x_1).$$

It turns out that λ_β is non-degenerate for any bounded Hamiltonian h [13]. Hence the equilibrium state is unique.

Denoting $k_\beta(x_0, x_1, x_2) = \lambda_\beta^{-1}\exp[-\beta h(x_0, x_1, x_2)]$, the characteristic function of $P_\beta^N(q)$ can be written as

$$\begin{aligned}
\widetilde{P_\beta^N}(k) &= \int \mathrm{d}q\, P_\beta^N(q)\exp(\mathrm{i}kq) \\
&= \sum_{x_0\ldots x_N, y_0 \ldots y_N} \exp\left(\frac{\mathrm{i}k}{2N+1}\sum_{i=-N}^{N} A(x_i \ldots x_{i+n})B(y_i \ldots y_{i+n})\right) \\
&\quad \times \phi_\beta(x_0, x_1)\phi_\beta(y_0, y_1)k_\beta(x_0, x_1, x_2)k_\beta(y_0, y_1, y_2) \\
&\quad \times \ldots k_\beta(x_{N-2}, x_{N-1}, x_N)k_\beta(y_{N-2}, y_{N-1}, y_N)\psi_\beta(x_{N-1}, x_N)\psi_\beta(y_{N-1}, y_N) \\
&= \sum_{x_0, x_1, y_0, y_1} \phi_\beta(x_0, x_1)\phi_\beta(y_0, y_1) \\
&\quad \times \exp\left(\frac{\mathrm{i}k}{2N+1}\sum_{i=-N}^{N} A(x_0 \ldots x_n)B(y_0 \ldots y_n)\right) \\
&\quad \times \left(T_\beta\left(\frac{k}{2N+1}\right)^{N-1}\xi_\beta\right)(x_0, y_0, x_1, y_1)
\end{aligned}$$

where $T_\beta(k)$ is an operator acting on functions $g: \{-1, 1\}^4 \to \mathbb{C}$ as

$$\begin{aligned}
(T_\beta(k)g)(x_0, y_0, x_1, y_1) &= \sum_{x_2, y_2} \exp[\mathrm{i}kA(x_0 \ldots x_n)B(y_0 \ldots y_n)] \\
&\quad \times k_\beta(x_0, x_1, x_2)k_\beta(y_0, y_1, y_2)g(x_1, y_1, x_2, y_2)
\end{aligned}$$

and $\xi_\beta(x_0, y_0, x_1, y_1) = \psi_\beta(x_0, x_1)\psi_\beta(y_0, y_1)$. We have to find the $\lim_{N\to\infty} P_\beta^N(q)$, but it is a standard result in probability theory that a sequence of probability measures converges properly to a limiting probability measure if the corresponding sequence of characteristic functions converges pointwise to a function which is continuous at the origin. Furthermore, the limiting function is the characteristic function of the limiting probability measure [14].

All this holds for finite β. It is clear, however, that the zero-temperature limit of the equilibrium state is still a quasiproduct state. Moreover, it can be characterised by a (renormalised) transfer matrix L in the same way as the finite-temperature equilibrium states [11]. But whereas the largest eigenvalue of L_β is non-degenerate for any bounded Hamiltonian, this has to be checked for L in each specific model.

We will give L for the PUD model and check the non-degeneracy of its largest eigenvalue in § 4. In any case, the foregoing argument to determine $P_\beta(q)$ remains valid for $\beta=\infty$.

Hence we have to establish the existence of $\lim_{N\to\infty} \widetilde{P_\beta^N}(k)$ for $\beta\in[0,\infty]$. We will prove that a sufficient condition for this limit to exist is that the largest eigenvalue of the transfer matrix is non-degenerate. We omit the β dependence in the notations.

Lemma 1. Let $\mu(k)$ denote the largest eigenvalue of $T(k)$ and let $S(k)$ be the corresponding eigenprojection. If $\mu(0)$ is non-degenerate then

$$\lim_{N\to\infty}\left(\frac{T(k/(2N+1))}{\mu(k/(2N+1))}\right)^N = S(0).$$

Proof. Decompose $T(k)$ in its canonical form:

$$T(k)=\sum_{i=1}^{s}\lambda_i(k)S_i(k)+D_i(k)$$

where $\lambda_i(k)$, $i=1,\dots,s$ are the eigenvalues of $T(k)$, $S_i(k)$ the corresponding eigenprojections and $D_i(k)$ the eigennilpotents, satisfying

$$\begin{aligned}
&S_i(k)S_j(k)=\delta_{ij}S_i(k)\\
&D_i(k)D_j(k)=\delta_{ij}D_i(k)^2\\
&D_i(k)S_j(k)=S_i(k)D_j(k)=\delta_{ij}D_i(k)\\
&D_i^n(k)=0 \qquad \text{for } n \text{ large enough.}
\end{aligned}$$

Furthermore, the $\lambda_i(k)$ depend analytically on k. Therefore the largest eigenvalue $\lambda_1(k)$ is non-degenerate for k small enough since $\lambda_1(0)$ is non-degenerate, and hence $D_1(k)=0$. All these results are standard (see, e.g., [15]). Now take k fixed and N large enough such that there exists an $n<N$ such that $D_i(k)^n=0, 1<i\leqslant s$ and $D_1(k/(2N+1))=0$, then

$$\begin{aligned}
T\left(\frac{k}{2N+1}\right)^N &= \sum_{0\leqslant p<n}\binom{N}{p}\sum_{i=1}^{s}\lambda_i\left(\frac{k}{2N+1}\right)^{N-p}S_i\left(\frac{k}{2N+1}\right)D_i\left(\frac{k}{2N+1}\right)^p\\
&=\lambda_1\left(\frac{k}{2N+1}\right)^N S\left(\frac{k}{2N+1}\right)\\
&\quad+\sum_{0\leqslant p<n}\binom{N}{p}\sum_{i=2}^{s}\lambda_i\left(\frac{k}{2N+1}\right)^{N-p}S_i\left(\frac{k}{2N+1}\right)D_i\left(\frac{k}{2N+1}\right)^p.
\end{aligned}$$

Since

$$\lim_{N\to\infty}\binom{N}{p}\left(\frac{\lambda_i(k/(2N+1))}{\lambda_1(k/(2N+1))}\right)^{N-p}=0$$

for $i\neq 1$, the result follows.

Theorem 2. If the largest eigenvalue of the transfer matrix is non-degenerate, then

$$P_\beta^N(q)=\sum_{xy}\delta(q-q_{xy}^N)\omega_\beta(x)\omega_\beta(y)$$

with

$$q_{xy}^{N}=\frac{1}{2N+1}A(x_i,\ldots,x_{i+n})B(y_i,\ldots,y_{i+n})$$

converges properly to the Dirac measure as $N\to\infty$, i.e. $P_\beta(q)=\delta(q-Q(\beta))$, concentrated in $Q(\beta)=\omega_\beta(A)\omega_\beta(B)$. This also holds for $\beta=\infty$ with $\omega_\infty=\lim_{\beta\to\infty}\omega_\beta$.

Proof. Since $T_\beta(0)=\lambda_\beta^{-2}L_\beta\otimes L_\beta$ (where $L_\infty=L$) and since by assumption the largest eigenvalue of L_β and hence of $T_\beta(0)$ is non-degenerate, we obtain from the lemma that

$$\lim_{N\to\infty}\widetilde{P_\beta^N}(k)=c\lim_{N\to\infty}\mu_\beta\left(\frac{k}{2N+1}\right)^N$$

$$=c\exp\left(k\lim_{\varepsilon\to 0}\frac{\log\mu_\beta(\varepsilon)}{\varepsilon}\right)=c\exp(k\mu'_\beta(0)).$$

Since this must be a characteristic function, we find that $c=1$ and that $\mu'_\beta(0)$ is purely imaginary. We conclude that $P_\beta(q)$ has a Dirac distribution concentrated at $Q(\beta)=-\mathrm{i}\mu'_\beta(0)$ or

$$Q(\beta)=\int \mathrm{d}q\, qP_\beta(q)=\lim_{N\to\infty}\sum_{xy}\int \mathrm{d}q\, q\delta(q-q_{xy}^N)\omega_\beta(x)\omega_\beta(y)$$

$$=\lim_{N\to\infty}\sum_{xy}q_{xy}^N\omega_\beta(x)\omega_\beta(y)$$

$$=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{i=-N}^{N}\omega_\beta(\tau_iA)\omega_\beta(\tau_iB)=\omega_\beta(A)\omega_\beta(B).$$

4. Application to the PUD model

In this section, we calculate explicitly $Q(\beta)$ for the model explained in § 2. Representing the function $\phi(x_0,x_1)$ as a four-component vector,

$$\phi=\begin{pmatrix}\phi(1,1)\\ \phi(1,-1)\\ \phi(-1,1)\\ \phi(-1,-1)\end{pmatrix}$$

the transfer matrix has the following form:

$$L_\beta=\begin{bmatrix}\mathrm{e}^{\beta/2} & 0 & \mathrm{e}^{-\beta/2} & 0\\ \mathrm{e}^{3\beta/2} & 0 & \mathrm{e}^{-3\beta/2} & 0\\ 0 & \mathrm{e}^{-3\beta/2} & 0 & \mathrm{e}^{3\beta/2}\\ 0 & \mathrm{e}^{-\beta/2} & 0 & \mathrm{e}^{\beta/2}\end{bmatrix}.$$

Its largest eigenvalue is given by

$$\lambda_\beta=\tfrac{1}{2}\mathrm{e}^{\beta/2}[1+\mathrm{e}^{-2\beta}+(5-2\,\mathrm{e}^{-2\beta}+\mathrm{e}^{-4\beta})^{1/2}]$$

and it is non-degenerate. The corresponding eigenvectors of L_β and L_β^* are respectively given by

$$\phi_\beta = \begin{pmatrix} e^{-\beta}\sigma_\beta \\ 1 \\ 1 \\ e^{-\beta}\sigma_\beta \end{pmatrix} \qquad \text{and} \qquad \psi_\beta = \frac{1}{2+2\sigma_\beta^2}\begin{pmatrix} e^{\beta}\sigma_\beta \\ 1 \\ 1 \\ e^{\beta}\sigma_\beta \end{pmatrix}$$

where $\sigma_\beta = (e^{-\beta/2}\lambda_\beta - 1)^{-1}$. The equilibrium states ω_β are completely determined by the conditional expectation

$$g_\beta(x_0, x_1, x_2) \equiv \frac{\rho_\beta(x_0, x_1, x_2)}{\rho'_\beta(x_0, x_1)} = \frac{1}{\lambda_\beta}\frac{\psi_\beta(x_1, x_2)}{\psi_\beta(x_0, x_1)} \exp[-\beta h(x_0, x_1, x_2)]$$

together with $\rho'_\beta(x_0, x_1) = \phi_\beta(x_0, x_1)\psi_\beta(x_0, x_1)$. Their zero-temperature limits exist and are equal to

$$g_\infty(x_0, x_1, x_2) = \psi(x_1, x_2)/\psi(x_0, x_1) \qquad \text{and} \qquad \rho'_\infty(x_0, x_1) = \phi(x_0, x_1)\psi(x_0, x_1)$$

with

$$\phi = \begin{pmatrix} \sigma \\ 1 \\ 1 \\ \sigma \end{pmatrix} \qquad \text{and} \qquad \psi = \frac{1}{2+2\sigma^2}\begin{pmatrix} \sigma \\ 1 \\ 1 \\ \sigma \end{pmatrix}$$

where $\sigma = \lim_{\beta\to\infty} \sigma_\beta = 2/(-1+\sqrt{5})$. They are the eigenvectors corresponding to the largest eigenvalue 1 of L and L^* where

$$L = \frac{1}{\sigma}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

One can easily check that its largest eigenvalue is non-degenerate. Indeed, all elements of the fourth power of L are strictly positive. The Perron–Frobenius theorem then guarantees that the largest eigenvalue of L^4 is non-degenerate and, by the spectral mapping theorem, this also holds for L. (This is the way that the non-degeneracy of the largest eigenvalue of L_β is proven for an arbitrary bounded Hamiltonian.) So we can conclude that $P_\beta(q)$ is a delta distribution for all β including $\beta = \infty$.

The observables used in the overlap are $A(x_0, x_1) = B(x_0, x_1) = x_0 x_1$. This gives for $\omega_\beta(A)$:

$$\begin{aligned}\omega_\beta(A) &= \sum_{x_0, x_1} \phi_\beta(x_0, x_1)\psi_\beta(x_0, x_1)x_0 x_1 \\ &= \sum_{x_0, x_1} \rho'_\beta(x_0, x_1)x_0 x_1 = \frac{\sigma_\beta^2 - 1}{\sigma_\beta^2 + 1}\end{aligned}$$

and

$$Q(\beta) = \left(\frac{\sigma_\beta^2 - 1}{\sigma_\beta^2 + 1}\right)^2 = \frac{1 - 2\,e^{-2\beta} + e^{-4\beta}}{5 - 2\,e^{-2\beta} + e^{-4\beta}}.$$

This is the value of the overlap for any β, including $\beta = \infty$. One can check that $Q(\beta)$ is a continuous function of β, also at ∞.

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